



## Department of Physics, IIT-Kanpur

Time : 2 hrs.

PhD Admission Test Dec 2019

Total Marks: 70

### Question 1

(a) Evaluate the following integral:

$$\oint_C \frac{e^{iz}}{z(z - \pi)} dz$$

for each of the following contours:

[5]

(i)  $C$  is a circle of radius  $R$  where  $R > \pi$ .

(ii)  $C$  is a circle of radius  $R$  where  $R < \pi$ .

(b) Radioactive nuclei decay according to the law  $\frac{dN}{dt} = -\lambda N$ , with  $N$  and  $\lambda$  being the concentration and the decay constant of a given nuclide, respectively. In a radioactive series of two different nuclides, with concentrations  $N_1(t)$  and  $N_2(t)$ , we have

$$\frac{dN_1}{dt} = -\lambda_1 N_1, \quad \text{and} \quad \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2.$$

Find  $N_2(t)$  for the initial conditions  $N_1(0) = N_0$  and  $N_2(0) = 0$ .

[5]

### Question 2

Fermion may occupy a given single-particle state with maximum occupation probability 1. A parafermion is a particle for which the maximum occupancy of any given single-particle state is  $k$ , which is a positive integer. Consider a system with one single-particle level whose energy is  $\epsilon$ , i.e. the total energy  $E = \epsilon N$ , where  $N$  is the particle number. Consider the system to be coupled with a reservoir of chemical potential  $\mu$  and temperature  $T$ .

(a) Write down the the grand-partition function and find an expression of the average occupation number  $\bar{N}$ .

[5]

(b) Considering  $k = 3$ , plot  $\bar{N}$  as function of  $\epsilon$ , clearly mentioning its value at  $\epsilon = \mu$ ,  $\epsilon \gg \mu$  and  $\epsilon \ll \mu$ .

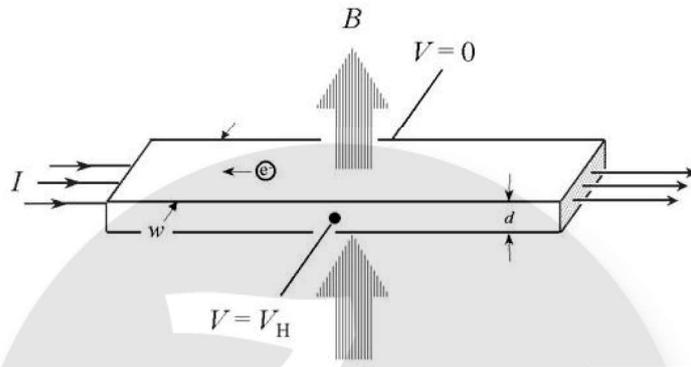
[5]

### Question 3

(a) Consider the schematic given below demonstrating Hall effect. Write down the expression for Hall voltage ( $V_H$ ) in terms of the applied current ( $I$ ) and the magnetic field ( $B$ ). If you want to design a Hall probe to measure an unknown magnetic field, would you use a metal



or semiconductor as the sensor? Whether a thin or thick sensor is desirable to enhance the sensitivity of the Hall probe? Justify your answers. [2+1+1]

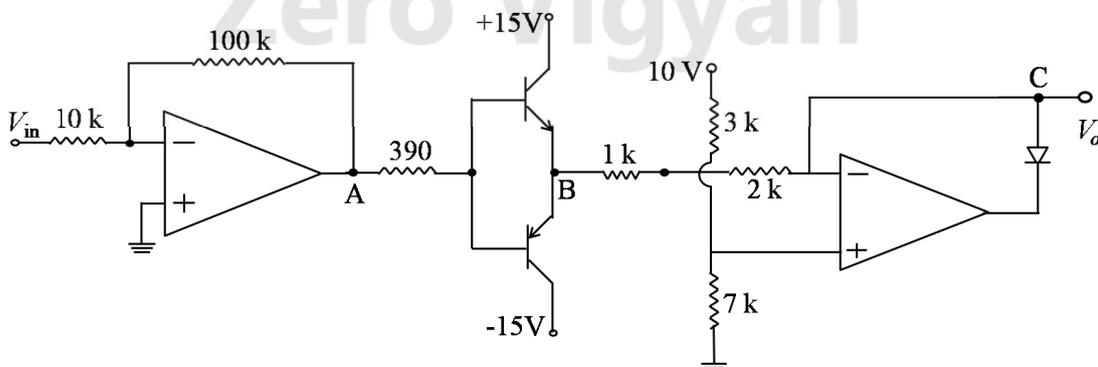


(b) What is the angular dispersion and resolving power of a diffraction grating? Would you use a grating or a prism for spectral analysis? Explain why. [2+1]

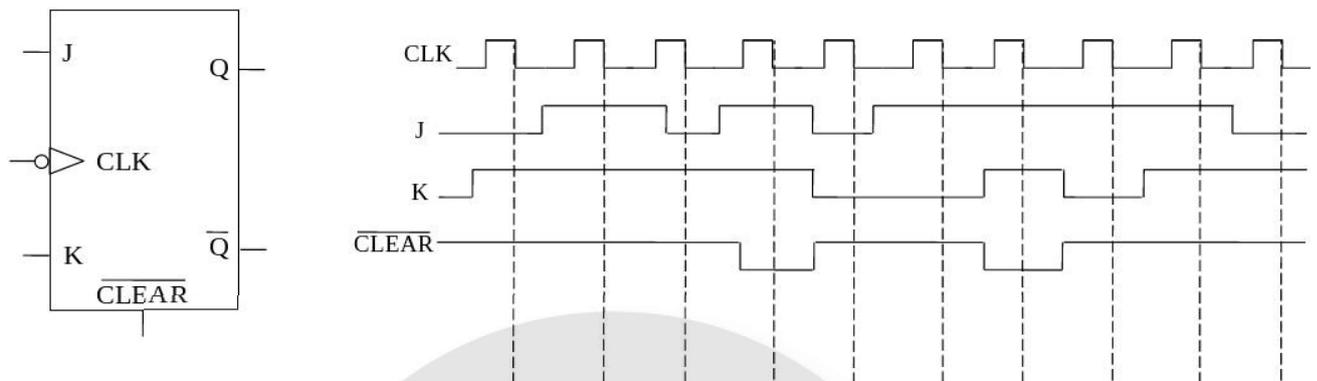
(c) What is the advantage of 4-probe measurement technique over 2-probe method? In which case the advantage becomes insignificant? [2+1]

### Question 4

(a) Sine wave of 1 V amplitude and 25 kHz frequency has been applied at the input  $V_{in}$  of the circuit shown in the figure. Draw waveforms corresponding to nodes A, B and C along with the  $V_{in}$  input waveform. [5]



(b) Draw the time diagram for Q output of the shown negative edge triggered J-K flip-flop for the given input pulses. Assume that the flip-flop was SET initially. [5]



### Question 5

Sketch a two-dimensional triangular lattice neatly and clearly. Assume that the length of each side of the triangle is  $d$ . Draw a primitive unit cell. Write down the basis vectors required to reproduce the whole crystal in 2D space and show them on the sketch. Calculate the reciprocal lattice vectors. [1+1+4+4]

### Question 6

(a) The Lagrangian of a free particle of mass  $m$  moving with velocity  $\mathbf{v}$ , in the inertial frame  $K$ , is  $L = \frac{1}{2}m\mathbf{v}^2$ . Suppose another reference frame  $K'$  moves with a velocity  $\mathbf{V}(t)$  with respect to frame  $K$ . Write down the Lagrangian  $L'$  in frame  $K'$ . Explain the dynamical origin of all the terms appearing in  $L'$ , and specify whether you expect such a Lagrangian from your knowledge of pseudo-forces. [5]

(b) There is a phase space transformation given as

$$Q = \alpha q^a p, \quad P = \beta q^b,$$

where  $a, b, \alpha, \beta$  are constants. Find out the values of  $a$  and  $b$  in terms of  $\alpha$  and  $\beta$  if the above transformation is a canonical transformation. [5]

### Question 7

(a) Consider an electron from rest is accelerated through a potential difference  $V$ . What is the de Broglie wavelength of the accelerated electron? (mass of the electron is  $m$  and charge  $e$ ). [2]

(b) Suppose  $|\alpha\rangle$  and  $|\beta\rangle$  are two parity eigenstates. Show that  $\langle\beta|\vec{r}|\alpha\rangle = 0$  unless the two states are of opposite parity. [3]





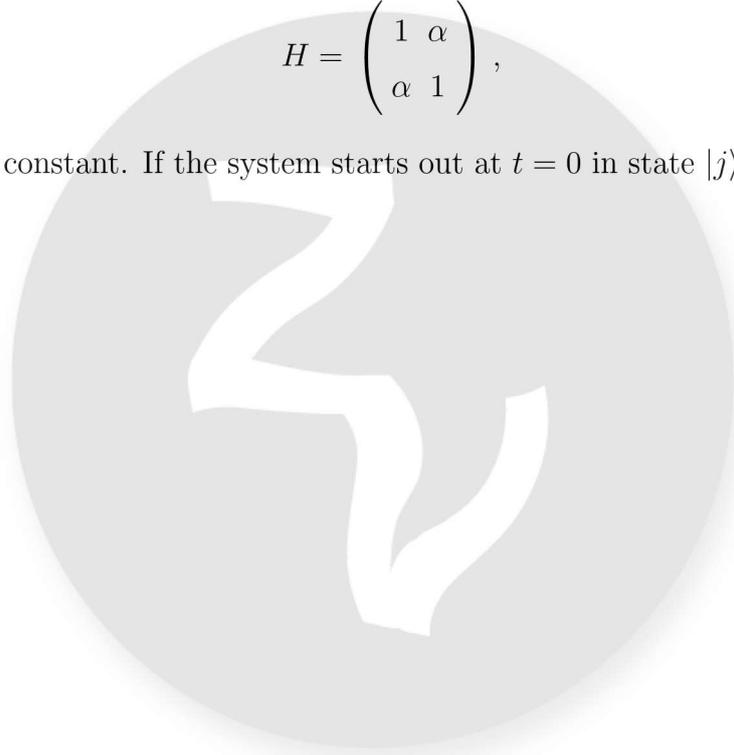
(c) Consider a two dimensional state space spanned by the basis states

$$|i\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |j\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Suppose the Hamiltonian of a system is given as

$$H = \begin{pmatrix} 1 & \alpha \\ \alpha & 1 \end{pmatrix},$$

where  $\alpha$  is a real constant. If the system starts out at  $t = 0$  in state  $|j\rangle$ , what is its state at time  $t$ . [5]



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