Department of Physics, IIT-Kanpur

Time : 2 hrs. **PhD Admission Test May 2015** **Total Marks: 100**

1a) Consider the states of the hydrogen atom $|n, l, m\rangle$. Find out the values of l, m for which the matrix element given by $\langle 3, l, m|z|3, 1, 1\rangle$ are nonzero, stating the reason. [6]

1b)Find the uncertainty in the position for a particle in the harmonic oscillator ground state. [6]

1c) Consider the state of a spin-1/2 particle, $|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$, where $|\uparrow\rangle$ and $|\downarrow\rangle$ are eigenstates of S^z with eigenvalues $\pm\hbar/2$ respectively. A stream of spin-1/2 particles in the above state pass through a Stern-Gerlach apparatus with the magnetic field along x direction. What is the fraction f_+ of the flux that emerge with $S^x = +\hbar/2$. Now, this flux of particles is further analysed by a Stern-Gerlach apparatus with the magnetic field along z direction. What is the fraction of the flux that emerge with $S^z = -\hbar/2$. [5+3]

2a) The region between two conducting plates is filled with a material of conductivity $1\mu\Omega^{-1}cm^{-1}$ and dielectric permittivity $\epsilon = 10\epsilon_0$ respectively ($\epsilon_0 = 8.85 \times 10^{-12}$ F/m being the free space permittivity). The resistance between the two conducting plates are given as $100k\Omega$. The two plates are connected to a battery of voltage 2 Volts. Calculate the value of the capacitance between the plates. [5]

2b) Existence of isolated magnetic monopoles may be conclusively established in future! Write down the modified Maxwell's equations in vacuum (In SI units) by including magnetic charge density which is a function of both space and time. [5]

2c) Assume that plane waves propagate in a non-permeable, anisotropic, source free dielectric medium. The dielectric is characterized by a tensor ϵ_{ij} but if coordinate axes are chosen as the principle axes, the components of displacement vector along these axes are related to the electric-field ($\vec{E}$) components by $D_i = \epsilon_i E_i (i = 1, 2, 3)$, where ϵ_i are the eigenvalues of the matrix ϵ_{ij}. Show that plane waves with frequency ω and wave vector $\vec{k} = k\hat{n}$ ($\hat{n}$ being the unit vector) must satisfy the following relation $\vec{k} \times (\vec{k} \times \vec{E}) + \mu_0\omega^2\vec{D} = 0$. Under what condition is the electric field $\vec{E}$ perpendicular to the wave vector $\vec{k}$? [10]

3a) Using Frobenius method obtain series solutions of the following differential equation:

$$\frac{d^2y}{dx} + x\frac{dy}{dx} + y = 0$$

[10]

3b) Calculate the following integral for **both** positive as well as negative values of k:

$$\int_{-\infty}^{\infty} \frac{\cos(kx)}{x^2+a^2}dx,$$

where a is a real positive constant. [10]

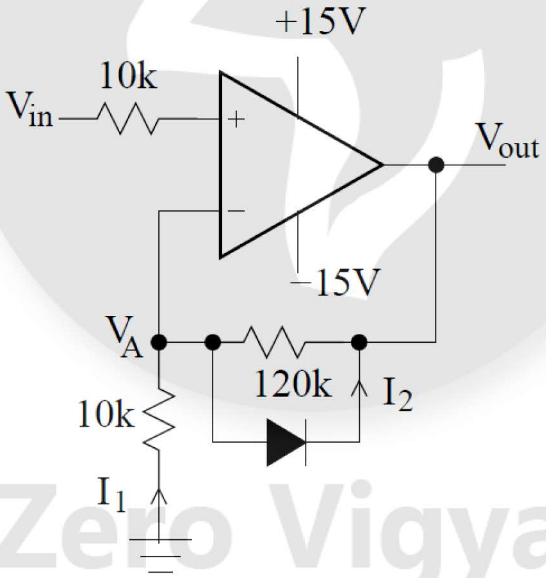
4) Consider in a *three-dimensional Euclidean space*, a simple pendulum consisting of an unstretchable string of length l carrying a point mass M that oscillates strictly in $x-z$ plane. Acceleration due to gravity is $\mathbf{g} = -g\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector along z-axis. The pendulum's point of support carries a point mass m restricted to move freely along x-axis. For this system, write down :

a) *all* the equations of constraints, [5]

b) number of degrees of freedom, [2]

c) the Euler-Lagrange equations, [8]

d) the generalised momenta. [5]



5a) In the op-amp circuit shown in the figure, determine V_{out}, V_A, I_1 and I_2 when (i) $V_{in} = 0.5$V and (ii) $V_{in} = -1.0$V. [5+5]

5b) (i) The diameter of a thin wire is measured to be 2 mm using a screw-gauge with 0.01 mm least count. The wire length is found to be 20 cm using a ruler with 1 mm least count. Find the volume of this wire in cc with error. [5]

(ii) Draw the schematic diagram of a photoelectric effect experiment and plot how the photocurrent varies as function of the voltage between the anode and the cathode. [5]